MAT 315 Take-Home Test #3

20 points with 2 extra credit points

A. **You must submit this exam by email to profwladis@gmail.com** (either type it or scan it in and send as a pdf file).

B. SHOW ALL WORK, and EXPLAIN all steps clearly with detailed steps. CLEARLY LABEL all problems.

C. You may discuss these problems with others, but your final answers must be ENTIRELY YOUR OWN WORDS.

1. [1 pt] Give an **orthonormal** basis $\mathcal{B}$ for $\mathbb{R}^4$ which is **NOT** the standard basis.
   a. [1 pt] Write the vector $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 5 \end{bmatrix}$ in terms of $\mathcal{B}$.

2. [1 pt] Draw an **orthogonal** basis for $\mathbb{R}^2$ in the 2-dimensional coordinate plane which is NOT a multiple of the standard basis.

3. [0.5 pt] Choose two distinct eigenvalues λ_1, λ_2 and choose a linearly **independent** set of three vectors S = {$\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$} in $\mathbb{R}^3$.
   a. [1.5 pts] Create a 3×3 matrix A with λ_1, λ_2 as eigenvalues and $\mathbf{v_1}$, $\mathbf{v_2}$, $\mathbf{v_3}$ as eigenvectors. Explain clearly and **show step-by-step** how you obtained A. (Hint: use the theory of matrix diagonalization.)
   b. [1.5 pts] Describe **each** eigenspace of A in terms of its **dimension** AND **placement in $\mathbb{R}^3$**. (Hint: You may want to use equations and/or to **draw** the eigenspaces in $\mathbb{R}^3$, or you may want to describe in words the visual placement of these eigenspaces in $\mathbb{R}^3$. i.e. Is the eigenspace a point, a line, a plane? Where in $\mathbb{R}^3$ is it located?)

4. [0.5 pt] Choose a linearly **dependent** set S that spans $\mathbb{R}^5$.
   a. [1 pt] Is S a basis for $\mathbb{R}^5$? **Explain WHY or WHY NOT**. If it is not a basis, **explain what must be done to turn it into a basis** for $\mathbb{R}^5$.
   b. [1 pt] Write the vector $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \\ -1 \end{bmatrix}$ as a linear combination of the vectors in the set S.
   c. [1 pt] Is the expression in part b. unique? **Explain WHY or WHY NOT.**

5. [1 pt] Choose a subspace H of $\mathbb{R}^4$ that does NOT contain any of the standard basis vectors for $\mathbb{R}^4$.
   a. [1 pt] Prove that H is a subspace of $\mathbb{R}^4$.

6. [1 pt] Choose a single vector **v** in $\mathbb{R}^2$ and select a linear transformation acting on $\mathbb{R}^2$ which will have **v** as an eigenvector.
    a. [1 pt] Draw a picture in the the 2-dimensional coordinate plane which shows the action of this linear transformation on the unit square (Hint: see pages 85-87 in the book for some examples of how to draw these kinds of pictures).
    b. [1 pt] Give a matrix which represents this linear transformation.
    c. [1 pt] Find the corresponding eigenvalue of **v** and **<u>explain</u>** its **<u>relationship</u>** to the **ACTION** of **A on v**.
    d. [1 pt] Does A have any other eigenvectors? If yes, **give their vector expression** AND **draw them** on the 2-D coordinate plane. If not, **EXPLAIN why not**.
    e. [1 pt] Is this linear transformation invertible? **EXPLAIN why or why not.**

7. [1 pt] Create a **NON-invertible** 4×4 matrix A and explain **WHY** it is not invertible.
    a. [1 pt] Give the **parametric vector form** of the solution set of A**x** = **0**.
    b. [1 pt] Give a **basis** for Nul A and give the **dimension** of Nul A.
    c. [1 pt] Give a **basis** for Col A and give the **dimension** of Col A.